

甲除奇乗表

某甲を実とし、某甲幕を法として帰除綴術にこれを除き、某甲で1を割った数を得る。

(一点鎖線の中は、私のメモです)

「某甲を実とし、某甲幕を法として帰除綴術にこれを除」くとは、分数式の級数展開を行うことを言います。

まず、某甲や某甲²についてもう一度はっきりさせておきます。

甲表起原の最初の部分(まだ式の番号も書いてない部分)から、

$$\text{径}^2 - \text{径}^2 \times \text{天}^2 = \text{某甲}^2 \quad \dots (0)$$

平方綴術にこれを開くと

(平方綴術に開く細かい計算は甲表起原で書いてますのでそれを見て下さい)

$$\text{径} - \frac{\text{径} \times \text{天}^2}{2} - \frac{\text{径} \times \text{天}^4}{8} - \frac{3 \times \text{径} \times \text{天}^6}{48} - \frac{15 \times \text{径} \times \text{天}^8}{384} = \text{某甲} \quad \dots (1)$$

$$\frac{1}{\text{某甲}^2} = \frac{1}{\text{径}^2} \times \frac{1}{1 - \text{天}^2} = \frac{1}{\text{径}^2} \times \frac{1 - \text{天}^2 + \text{天}^2}{1 - \text{天}^2} = \frac{1}{\text{径}^2} \times \left(1 + \frac{\text{天}^2}{1 - \text{天}^2} \right)$$

$$\frac{\text{径}^2}{\text{某甲}^2} = \frac{1}{1 - \text{天}^2} = \frac{1 - \text{天}^2 + \text{天}^2}{1 - \text{天}^2} = 1 + \frac{\text{天}^2}{1 - \text{天}^2} = 1 + \text{天}^2 \times \frac{\text{径}^2}{\text{某甲}^2}$$

$$= 1 + \text{天}^2 \times \frac{1 - \text{天}^2 + \text{天}^2}{1 - \text{天}^2} = 1 + \text{天}^2 \times \left(1 + \frac{\text{天}^2}{1 - \text{天}^2} \right)$$

これを繰り返していけば級数の形にできる。

求めようとしているのは、 $\frac{1}{\text{某甲}}$ なので、

$$\frac{1}{\text{某甲}} = \frac{\text{某甲}}{\text{某甲}^2} = \frac{\text{某甲}}{\text{径}^2} \times \frac{\text{径}^2}{\text{某甲}^2} = \frac{\text{某甲}}{\text{径}^2} \times \left[1 + \text{天}^2 \times \left(1 + \frac{\text{天}^2}{1 - \text{天}^2} \right) \right]$$

$$= \frac{\text{某甲}}{\text{径}^2} \times \left[1 + \text{天}^2 \times \left\{ 1 + \text{天}^2 \times \left(1 + \frac{\text{天}^2}{1 - \text{天}^2} \right) \right\} \right]$$

$$= \frac{\text{某甲}}{\text{径}^2} \times \left[1 + \text{天}^2 + \text{天}^4 \times \left(1 + \frac{\text{天}^2}{1 - \text{天}^2} \right) \right]$$

つづく

前ページからのつづき

$$\begin{aligned} \frac{1}{\text{某甲}} &= \frac{\text{某甲}}{\text{径}^2} \times \left[1 + \text{天}^2 + \text{天}^4 \times \left\{ 1 + \text{天}^2 \times \left(1 + \frac{\text{天}^2}{1 - \text{天}^2} \right) \right\} \right] \\ &= \frac{\text{某甲}}{\text{径}^2} \times \left[1 + \text{天}^2 + \text{天}^4 \times \left\{ 1 + \text{天}^2 \times \left\{ 1 + \text{天}^2 \times \left(1 + \frac{\text{天}^2}{1 - \text{天}^2} \right) \right\} \right\} \right] \\ &= \frac{\text{某甲}}{\text{径}^2} \times \left[1 + \text{天}^2 + \text{天}^4 + \text{天}^6 \times \left\{ 1 + \text{天}^2 \times \left(1 + \frac{\text{天}^2}{1 - \text{天}^2} \right) \right\} \right] \\ &= \frac{\text{某甲}}{\text{径}^2} \times \left[1 + \text{天}^2 + \text{天}^4 + \text{天}^6 + \text{天}^8 \times \left(1 + \frac{\text{天}^2}{1 - \text{天}^2} \right) \right] \\ &= \frac{\text{某甲}}{\text{径}^2} \times \left[1 + \text{天}^2 + \text{天}^4 + \text{天}^6 + \text{天}^8 + \text{天}^8 \times \frac{\text{天}^2}{1 - \text{天}^2} \right] \end{aligned}$$

天の9乗以上は無視して

$$\frac{1}{\text{某甲}} = \frac{\text{某甲}}{\text{径}^2} \times \left[1 + \text{天}^2 + \text{天}^4 + \text{天}^6 + \text{天}^8 \right] \quad \dots (2)$$

(2) の分子の某甲に、(1) を代入して

$$\begin{aligned} \frac{1}{\text{某甲}} &= \frac{1}{\text{径}^2} \times \left(\text{径} - \frac{\text{径} \times \text{天}^2}{2} - \frac{\text{径} \times \text{天}^4}{8} - \frac{3 \times \text{径} \times \text{天}^6}{48} - \frac{15 \times \text{径} \times \text{天}^8}{384} \right) \\ &\quad \times \left(1 + \text{天}^2 + \text{天}^4 + \text{天}^6 + \text{天}^8 \right) \\ \frac{1}{\text{某甲}} &= \frac{1}{\text{径}} \times \left(1 - \frac{\text{天}^2}{2} - \frac{\text{天}^4}{8} - \frac{3 \times \text{天}^6}{48} - \frac{15 \times \text{天}^8}{384} \right) \\ &\quad \times \left(1 + \text{天}^2 + \text{天}^4 + \text{天}^6 + \text{天}^8 \right) \end{aligned}$$

ここの計算も、天の9乗以上は無視する。

$$\begin{aligned} \frac{1}{\text{某甲}} &= \frac{1}{\text{径}} \times \left(1 - \frac{\text{天}^2}{2} - \frac{\text{天}^4}{8} - \frac{3 \times \text{天}^6}{48} - \frac{15 \times \text{天}^8}{384} + \text{天}^2 - \frac{\text{天}^4}{2} - \frac{\text{天}^6}{8} - \frac{3 \times \text{天}^8}{48} \right. \\ &\quad \left. + \text{天}^4 - \frac{\text{天}^6}{2} - \frac{\text{天}^8}{8} + \text{天}^6 - \frac{\text{天}^8}{2} + \text{天}^8 \right) \end{aligned}$$

つづく

前ページからのつづき

$$\frac{1}{\text{某甲}} = \frac{1}{\text{径}} \times \left(1 - \frac{\text{天}^2}{2} - \frac{\text{天}^4}{8} - \frac{3 \times \text{天}^6}{48} - \frac{15 \times \text{天}^8}{384} + \text{天}^2 - \frac{\text{天}^4}{2} - \frac{\text{天}^6}{8} - \frac{3 \times \text{天}^8}{48} \right. \\ \left. + \text{天}^4 - \frac{\text{天}^6}{2} - \frac{\text{天}^8}{8} + \text{天}^6 - \frac{\text{天}^8}{2} + \text{天}^8 \right)$$

$$\frac{1}{\text{某甲}} = \frac{1}{\text{径}} \times \left(1 + \text{天}^2 - \frac{\text{天}^2}{2} + \text{天}^4 - \frac{\text{天}^4}{2} - \frac{\text{天}^4}{8} + \text{天}^6 - \frac{\text{天}^6}{2} - \frac{\text{天}^6}{8} - \frac{3 \times \text{天}^6}{48} \right. \\ \left. + \text{天}^8 - \frac{\text{天}^8}{2} - \frac{\text{天}^8}{8} - \frac{15 \times \text{天}^8}{384} \right)$$

$$\frac{1}{\text{某甲}} = \frac{1}{\text{径}} \times \left(1 + \frac{\text{天}^2}{2} + \frac{3 \times \text{天}^4}{8} + \frac{3 \times 5 \times \text{天}^6}{48} + \frac{3 \times 5 \times 7 \times \text{天}^8}{384} \right)$$

両辺に 径 をかけて

$$1 + \frac{\text{天}^2}{2} + \frac{3 \times \text{天}^4}{8} + \frac{15 \times \text{天}^6}{48} + \frac{105 \times \text{天}^8}{384} = \frac{\text{径}}{\text{某甲}} \quad \dots (3)$$

(3) の両辺に 天 をかけて

$$\text{天} + \frac{\text{天}^3}{2} + \frac{3 \times \text{天}^5}{8} + \frac{15 \times \text{天}^7}{48} + \frac{105 \times \text{天}^9}{384} = \frac{\text{径} \times \text{天}}{\text{某甲}} \quad \dots (4)$$

(4) の両辺に 天² をかけて

$$\text{天}^3 + \frac{\text{天}^5}{2} + \frac{3 \times \text{天}^7}{8} + \frac{15 \times \text{天}^9}{48} + \frac{105 \times \text{天}^{11}}{384} = \frac{\text{径} \times \text{天}^3}{\text{某甲}} \quad \dots (5)$$

(5) の両辺に 天² をかけて

$$\text{天}^5 + \frac{\text{天}^7}{2} + \frac{3 \times \text{天}^9}{8} + \frac{15 \times \text{天}^{11}}{48} + \frac{105 \times \text{天}^{13}}{384} = \frac{\text{径} \times \text{天}^5}{\text{某甲}} \quad \dots (6)$$

(6) の両辺に 天² をかけて

$$\text{天}^7 + \frac{\text{天}^9}{2} + \frac{3 \times \text{天}^{10}}{8} + \frac{15 \times \text{天}^{12}}{48} + \frac{105 \times \text{天}^{14}}{384} = \frac{\text{径} \times \text{天}^7}{\text{某甲}} \quad \dots (7)$$

それぞれの天累乗幂の畳数を解いて、天の累乗を某甲で割ったときの畳数を求める。

まず (4) で考える。

$$\text{天} + \frac{\text{天}^3}{2} + \frac{3 \times \text{天}^5}{8} + \frac{15 \times \text{天}^7}{48} + \frac{105 \times \text{天}^9}{384} = \frac{\text{径} \times \text{天}}{\text{某甲}} \quad \dots (4)$$

各項の畳数を求める。

左辺第1項の 天 の畳数は、天畳数 = $\frac{\text{截数}}{2}$ より $\frac{\text{截数}}{2}$

左辺第2項の $\frac{\text{天}^3}{2}$ の畳数は、天再畳数 = $\frac{\text{截数}}{4}$ より $\frac{\text{截数}}{2 \times 4}$

左辺第3項の $\frac{3 \times \text{天}^5}{8}$ の畳数は、天四畳数 = $\frac{\text{截数}}{6}$ より $\frac{3 \times \text{截数}}{8 \times 6}$

左辺第4項の $\frac{15 \times \text{天}^7}{48}$ の畳数は、天六畳数 = $\frac{\text{截数}}{8}$ より $\frac{15 \times \text{截数}}{48 \times 8}$

左辺第5項の $\frac{105 \times \text{天}^9}{384}$ の畳数は、天八畳数 = $\frac{\text{截数}}{10}$ より $\frac{105 \times \text{截数}}{384 \times 10}$

$$\frac{\text{截数}}{2} + \frac{\text{截数}}{2 \times 4} + \frac{3 \times \text{截数}}{8 \times 6} + \frac{15 \times \text{截数}}{48 \times 8} + \frac{105 \times \text{截数}}{384 \times 10} = \frac{\text{径} \times \text{天}}{\text{某甲}} \text{畳数} \quad \dots (8)$$

(8) の両辺を 径 で割って、(9) となる。

(5) ~ (7) にかんしても同様に計算して、(10) ~ (12) を得る。

$$\frac{\text{截数}}{2 \times \text{径}} + \frac{\text{截数}}{2 \times 4 \times \text{径}} + \frac{3 \times \text{截数}}{8 \times 6 \times \text{径}} + \frac{15 \times \text{截数}}{48 \times 8 \times \text{径}} + \frac{105 \times \text{截数}}{384 \times 10 \times \text{径}} = \frac{\text{天}}{\text{某甲}} \text{畳数} \quad \dots (9)$$

$$\frac{\text{截数}}{4 \times \text{径}} + \frac{\text{截数}}{2 \times 6 \times \text{径}} + \frac{3 \times \text{截数}}{8 \times 8 \times \text{径}} + \frac{15 \times \text{截数}}{48 \times 10 \times \text{径}} + \frac{105 \times \text{截数}}{384 \times 12 \times \text{径}} = \frac{\text{天}^3}{\text{某甲}} \text{畳数} \quad \dots (10)$$

$$\frac{\text{截数}}{6 \times \text{径}} + \frac{\text{截数}}{2 \times 8 \times \text{径}} + \frac{3 \times \text{截数}}{8 \times 10 \times \text{径}} + \frac{15 \times \text{截数}}{48 \times 12 \times \text{径}} + \frac{105 \times \text{截数}}{384 \times 14 \times \text{径}} = \frac{\text{天}^5}{\text{某甲}} \text{疊数}$$

... (11)

$$\frac{\text{截数}}{8 \times \text{径}} + \frac{\text{截数}}{2 \times 10 \times \text{径}} + \frac{3 \times \text{截数}}{8 \times 12 \times \text{径}} + \frac{15 \times \text{截数}}{48 \times 14 \times \text{径}} + \frac{105 \times \text{截数}}{384 \times 16 \times \text{径}} = \frac{\text{天}^7}{\text{某甲}} \text{疊数}$$

... (12)

このあと、前空数 が出て来るので、甲表起原で説明されている内容を再掲する。

1 - 1 = 0 (これを空積と呼ぶ)

平方綴術にこれを解いて得られた答えを 前空数 と名前をつける。

$$1 - \frac{1}{2} - \frac{1}{8} - \frac{3}{48} - \frac{15}{384} - \frac{105}{3840} = \text{前空数}$$

(以前よりも、項を1つ多くしたが特別変わった訳ではない。近似の程度が少し変わっただけです。)

$\frac{\text{截数}}{\text{径}} - \frac{\text{截数} \times \text{前空数}}{\text{径}}$ これを解くと

$$\frac{\text{截数}}{\text{径}} - \frac{\text{截数}}{\text{径}} \times \left(1 - \frac{1}{2} - \frac{1}{8} - \frac{3}{48} - \frac{15}{384} - \frac{105}{3840} \right)$$

$$= \frac{\text{截数}}{2 \times \text{径}} + \frac{\text{截数}}{8 \times \text{径}} + \frac{3 \times \text{截数}}{48 \times \text{径}} + \frac{15 \times \text{截数}}{384 \times \text{径}} + \frac{105 \times \text{截数}}{3840 \times \text{径}}$$

$$\frac{\text{截数}}{2 \times \text{径}} + \frac{\text{截数}}{2 \times 4 \times \text{径}} + \frac{3 \times \text{截数}}{8 \times 6 \times \text{径}} + \frac{15 \times \text{截数}}{48 \times 8 \times \text{径}}$$

これは、 $\frac{\text{天}}{\text{某甲}}$ 疊数 と全く同じなので

$$\frac{\text{截数}}{\text{径}} = \frac{\text{天}}{\text{某甲}} \text{疊数} \quad \dots (13)$$

$$\frac{2 \times \text{天}}{\text{某甲}} \text{暈数} - \frac{\text{截数} \times \text{前空数}}{\text{径}} \quad \text{これを解いて通分すると}$$

$$\begin{aligned} & 2 \times \left(\frac{\text{截数}}{2 \times \text{径}} + \frac{\text{截数}}{2 \times 4 \times \text{径}} + \frac{3 \times \text{截数}}{8 \times 6 \times \text{径}} + \frac{15 \times \text{截数}}{48 \times 8 \times \text{径}} + \frac{105 \times \text{截数}}{384 \times 10 \times \text{径}} \right) \\ & - \frac{\text{截数}}{\text{径}} \times \left(1 - \frac{1}{2} - \frac{1}{8} - \frac{3}{48} - \frac{15}{384} \right) \\ & = \left(\frac{\text{截数}}{4 \times \text{径}} + \frac{\text{截数}}{2 \times \text{径}} \right) + \left(\frac{3 \times \text{截数}}{4 \times 6 \times \text{径}} + \frac{\text{截数}}{8 \times \text{径}} \right) + \left(\frac{15 \times \text{截数}}{24 \times 8 \times \text{径}} + \frac{3 \times \text{截数}}{48 \times \text{径}} \right) \\ & \quad + \left(\frac{105 \times \text{截数}}{192 \times 10 \times \text{径}} + \frac{15 \times \text{截数}}{384 \times \text{径}} \right) \\ & = \frac{3 \times \text{截数}}{4 \times \text{径}} + \frac{3 \times \text{截数} + 3 \times \text{截数}}{4 \times 6 \times \text{径}} + \frac{15 \times \text{截数} + 12 \times \text{截数}}{24 \times 8 \times \text{径}} \\ & \quad + \frac{105 \times \text{截数} + 75 \times \text{截数}}{192 \times 10 \times \text{径}} \\ & = \frac{3 \times \text{截数}}{4 \times \text{径}} + \frac{2 \times 3 \times \text{截数}}{4 \times 6 \times \text{径}} + \frac{3 \times 9 \times \text{截数}}{24 \times 8 \times \text{径}} + \frac{4 \times 3 \times 15 \times \text{截数}}{192 \times 10 \times \text{径}} \end{aligned}$$

$$0 + \frac{3 \times \text{截数}}{4 \times \text{径}} + \frac{3 \times \text{截数}}{2 \times 6 \times \text{径}} + \frac{3 \times 3 \times \text{截数}}{8 \times 8 \times \text{径}} + \frac{3 \times 15 \times \text{截数}}{48 \times 10 \times \text{径}}$$

これは、 $\frac{\text{天}^3}{\text{某甲}}$ 暈数 の3倍と全く同じなので

$$\frac{2 \times \text{截数}}{3 \times \text{径}} = \frac{\text{天}^3}{\text{某甲}} \text{暈数} \quad \dots (14)$$

$$\frac{4 \times \text{天}^3}{\text{某甲}} \text{暈数} - \frac{\text{截数} \times \text{前空数}}{\text{径}} \quad \text{これを解いて通分すると}$$

$$\begin{aligned} & 4 \times \left(\frac{\text{截数}}{4 \times \text{径}} + \frac{\text{截数}}{2 \times 6 \times \text{径}} + \frac{3 \times \text{截数}}{8 \times 8 \times \text{径}} + \frac{15 \times \text{截数}}{48 \times 10 \times \text{径}} + \frac{105 \times \text{截数}}{384 \times 12 \times \text{径}} \right) \\ & - \frac{\text{截数}}{\text{径}} \times \left(1 - \frac{1}{2} - \frac{1}{8} - \frac{3}{48} - \frac{15}{384} \right) \\ & = \left(\frac{4 \times \text{截数}}{2 \times 6 \times \text{径}} + \frac{\text{截数}}{2 \times \text{径}} \right) + \left(\frac{12 \times \text{截数}}{8 \times 8 \times \text{径}} + \frac{\text{截数}}{8 \times \text{径}} \right) + \left(\frac{60 \times \text{截数}}{48 \times 10 \times \text{径}} + \frac{3 \times \text{截数}}{48 \times \text{径}} \right) \\ & \quad + \left(\frac{420 \times \text{截数}}{384 \times 12 \times \text{径}} + \frac{15 \times \text{截数}}{384 \times \text{径}} \right) \end{aligned}$$

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前ページのつづき

$$\begin{aligned}
 &= \frac{4 \times \text{截数} + 6 \times \text{截数}}{2 \times 6 \times \text{径}} + \frac{12 \times \text{截数} + 8 \times \text{截数}}{8 \times 8 \times \text{径}} + \frac{60 \times \text{截数} + 30 \times \text{截数}}{48 \times 10 \times \text{径}} \\
 &\quad + \frac{420 \times \text{截数} + 180 \times \text{截数}}{384 \times 12 \times \text{径}} \\
 &= \frac{2 \times 5 \times \text{截数}}{2 \times 6 \times \text{径}} + \frac{4 \times 5 \times \text{截数}}{8 \times 8 \times \text{径}} + \frac{6 \times 5 \times 3 \times \text{截数}}{48 \times 10 \times \text{径}} + \frac{8 \times 5 \times 15 \times \text{截数}}{384 \times 12 \times \text{径}}
 \end{aligned}$$

$$0 + \frac{5 \times \text{截数}}{6 \times \text{径}} + \frac{5 \times \text{截数}}{2 \times 8 \times \text{径}} + \frac{5 \times 3 \times \text{截数}}{8 \times 10 \times \text{径}} + \frac{5 \times 15 \times \text{截数}}{48 \times 12 \times \text{径}}$$

これは、 $\frac{\text{天}^5}{\text{某甲}}$ 量数の5倍と全く同じなので

$$\frac{2 \times 4 \times \text{截数}}{3 \times 5 \times \text{径}} = \frac{\text{天}^5}{\text{某甲}} \text{量数} \quad \dots (15)$$

$$\frac{6 \times \text{天}^5}{\text{某甲}} \text{量数} - \frac{\text{截数} \times \text{前空数}}{\text{径}} \quad \text{これを解いて通分すると}$$

$$\begin{aligned}
 &6 \times \left(\frac{\text{截数}}{6 \times \text{径}} + \frac{\text{截数}}{2 \times 8 \times \text{径}} + \frac{3 \times \text{截数}}{8 \times 10 \times \text{径}} + \frac{15 \times \text{截数}}{48 \times 12 \times \text{径}} + \frac{105 \times \text{截数}}{384 \times 14 \times \text{径}} \right) \\
 &- \frac{\text{截数}}{\text{径}} \times \left(1 - \frac{1}{2} - \frac{1}{8} - \frac{3}{48} - \frac{15}{384} \right) \\
 &= \left(\frac{6 \times \text{截数}}{2 \times 8 \times \text{径}} + \frac{\text{截数}}{2 \times \text{径}} \right) + \left(\frac{18 \times \text{截数}}{8 \times 10 \times \text{径}} + \frac{\text{截数}}{8 \times \text{径}} \right) + \left(\frac{90 \times \text{截数}}{48 \times 12 \times \text{径}} + \frac{3 \times \text{截数}}{48 \times \text{径}} \right) \\
 &\quad + \left(\frac{630 \times \text{截数}}{384 \times 14 \times \text{径}} + \frac{15 \times \text{截数}}{384 \times \text{径}} \right) \\
 &= \frac{6 \times \text{截数} + 8 \times \text{截数}}{2 \times 8 \times \text{径}} + \frac{18 \times \text{截数} + 10 \times \text{截数}}{8 \times 10 \times \text{径}} + \frac{90 \times \text{截数} + 36 \times \text{截数}}{48 \times 12 \times \text{径}} \\
 &\quad + \frac{630 \times \text{截数} + 210 \times \text{截数}}{384 \times 14 \times \text{径}} \\
 &= \frac{2 \times 7 \times \text{截数}}{2 \times 8 \times \text{径}} + \frac{4 \times 7 \times \text{截数}}{8 \times 10 \times \text{径}} + \frac{6 \times 7 \times 3 \times \text{截数}}{48 \times 12 \times \text{径}} + \frac{8 \times 7 \times 15 \times \text{截数}}{384 \times 14 \times \text{径}}
 \end{aligned}$$

$$0 + \frac{7 \times \text{截数}}{8 \times \text{径}} + \frac{7 \times \text{截数}}{2 \times 10 \times \text{径}} + \frac{7 \times 3 \times \text{截数}}{8 \times 12 \times \text{径}} + \frac{7 \times 15 \times \text{截数}}{48 \times 14 \times \text{径}}$$

これは、 $\frac{\text{天}^7}{\text{某甲}}$ 疊数 の7倍と全く同じなので

$$\frac{2 \times 4 \times 6 \times \text{截数}}{3 \times 5 \times 7 \times \text{径}} = \frac{\text{天}^7}{\text{某甲}} \text{疊数} \quad \dots (16)$$

このように、求めて、 甲除奇乗表 を作る

甲除偶乗表

1を 某甲 で割った数を置き、天² を累乗して、畳数を求める。

(一点鎖線の中は、私のメモです)

甲除奇乗表の(3)より

$$1 + \frac{\text{天}^2}{2} + \frac{3 \times \text{天}^4}{8} + \frac{15 \times \text{天}^6}{48} + \frac{105 \times \text{天}^8}{384} = \frac{\text{径}}{\text{某甲}} \quad \dots (3)$$

各項の畳数を求める。

左辺第1項の 1 の畳数は、截数

左辺第2項の $\frac{\text{天}^2}{2}$ の畳数は、天²畳数 = $\frac{\text{截数}}{3}$ より $\frac{\text{截数}}{2 \times 3}$

左辺第3項の $\frac{3 \times \text{天}^4}{8}$ の畳数は、天³畳数 = $\frac{\text{截数}}{5}$ より $\frac{3 \times \text{截数}}{8 \times 5}$

左辺第4項の $\frac{15 \times \text{天}^6}{48}$ の畳数は、天⁵畳数 = $\frac{\text{截数}}{7}$ より $\frac{15 \times \text{截数}}{48 \times 7}$

左辺第5項の $\frac{105 \times \text{天}^8}{384}$ の畳数は、天⁷畳数 = $\frac{\text{截数}}{9}$ より $\frac{105 \times \text{截数}}{384 \times 9}$

$$\text{截数} + \frac{\text{截数}}{2 \times 3} + \frac{3 \times \text{截数}}{8 \times 5} + \frac{15 \times \text{截数}}{48 \times 7} + \frac{105 \times \text{截数}}{384 \times 9} = \frac{\text{径}}{\text{某甲}} \text{畳数} \quad \dots (17)$$

両辺を 径 で割って

$$\frac{\text{截数}}{\text{径}} + \frac{\text{截数}}{2 \times 3 \times \text{径}} + \frac{3 \times \text{截数}}{8 \times 5 \times \text{径}} + \frac{15 \times \text{截数}}{48 \times 7 \times \text{径}} + \frac{105 \times \text{截数}}{384 \times 9 \times \text{径}} = \frac{1}{\text{某甲}} \text{畳数} \quad \dots (18)$$

(3)の両辺に 天² を掛けて

$$\text{天}^2 + \frac{\text{天}^4}{2} + \frac{3 \times \text{天}^6}{8} + \frac{15 \times \text{天}^8}{48} + \frac{105 \times \text{天}^{10}}{384} = \frac{\text{天}^2 \times \text{径}}{\text{某甲}} \quad \dots (19)$$

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(19) の両辺に 天² を掛けて

$$\text{天}^4 + \frac{\text{天}^6}{2} + \frac{3 \times \text{天}^8}{8} + \frac{15 \times \text{天}^{10}}{48} + \frac{105 \times \text{天}^{12}}{384} = \frac{\text{天}^4 \times \text{径}}{\text{某甲}} \quad \dots (20)$$

(20) の両辺に 天² を掛けて

$$\text{天}^6 + \frac{\text{天}^8}{2} + \frac{3 \times \text{天}^{10}}{8} + \frac{15 \times \text{天}^{12}}{48} + \frac{105 \times \text{天}^{14}}{384} = \frac{\text{天}^6 \times \text{径}}{\text{某甲}} \quad \dots (21)$$

(21) の両辺に 天² を掛けて

$$\text{天}^8 + \frac{\text{天}^{10}}{2} + \frac{3 \times \text{天}^{12}}{8} + \frac{15 \times \text{天}^{14}}{48} + \frac{105 \times \text{天}^{16}}{384} = \frac{\text{天}^8 \times \text{径}}{\text{某甲}} \quad \dots (22)$$

(19) ~ (22) の畳数を求め 径 で割り、(23) ~ (26) を得る。

$$\frac{\text{截数}}{3 \times \text{径}} + \frac{\text{截数}}{2 \times 5 \times \text{径}} + \frac{3 \times \text{截数}}{8 \times 7 \times \text{径}} + \frac{15 \times \text{截数}}{48 \times 9 \times \text{径}} + \frac{105 \times \text{截数}}{384 \times 11 \times \text{径}} = \frac{\text{天}^2 \times \text{径}}{\text{某甲}} \text{畳数} \quad \dots (23)$$

$$\frac{\text{截数}}{5 \times \text{径}} + \frac{\text{截数}}{2 \times 7 \times \text{径}} + \frac{3 \times \text{截数}}{8 \times 9 \times \text{径}} + \frac{15 \times \text{截数}}{48 \times 11 \times \text{径}} + \frac{105 \times \text{截数}}{384 \times 13 \times \text{径}} = \frac{\text{天}^4 \times \text{径}}{\text{某甲}} \text{畳数} \quad \dots (24)$$

$$\frac{\text{截数}}{7 \times \text{径}} + \frac{\text{截数}}{2 \times 9 \times \text{径}} + \frac{3 \times \text{截数}}{8 \times 11 \times \text{径}} + \frac{15 \times \text{截数}}{48 \times 13 \times \text{径}} + \frac{105 \times \text{截数}}{384 \times 15 \times \text{径}} = \frac{\text{天}^6 \times \text{径}}{\text{某甲}} \text{畳数} \quad \dots (25)$$

$$\frac{\text{截数}}{9 \times \text{径}} + \frac{\text{截数}}{2 \times 11 \times \text{径}} + \frac{3 \times \text{截数}}{8 \times 13 \times \text{径}} + \frac{15 \times \text{截数}}{48 \times 15 \times \text{径}} + \frac{105 \times \text{截数}}{384 \times 17 \times \text{径}} = \frac{\text{天}^8 \times \text{径}}{\text{某甲}} \text{畳数} \quad \dots (26)$$

甲表起原の(4)式より

$$1 - \frac{1}{2 \times 3} - \frac{1}{8 \times 5} - \frac{3}{48 \times 7} - \frac{15}{384 \times 9} = \frac{\text{円積}}{\text{径}^2} = \text{円積率} \quad \dots (4)$$

したがって

$$1 - \frac{1}{2 \times 3} - \frac{1}{8 \times 5} - \frac{3}{48 \times 7} - \frac{15}{384 \times 9} = \text{円積率} \quad \dots (27)$$

$$\frac{2 \times \text{円積率} \times \text{截数}}{\text{径}} - \frac{\text{截数} \times \text{前空数}}{\text{径}} \quad \text{これを解くと}$$

$$\begin{aligned} & \frac{2 \times \text{截数}}{\text{径}} \times \left(1 - \frac{1}{2 \times 3} - \frac{1}{8 \times 5} - \frac{3}{48 \times 7} - \frac{15}{384 \times 9} \right) \\ & - \frac{\text{截数}}{\text{径}} \times \left(1 - \frac{1}{2} - \frac{1}{8} - \frac{3}{48} - \frac{15}{384} - \frac{105}{3840} \right) \\ & = \frac{\text{截数}}{\text{径}} + \left(\frac{\text{截数}}{2 \times \text{径}} - \frac{\text{截数}}{3 \times \text{径}} \right) + \left(\frac{\text{截数}}{8 \times \text{径}} - \frac{2 \times \text{截数}}{8 \times 5 \times \text{径}} \right) + \left(\frac{3 \times \text{截数}}{48 \times \text{径}} - \frac{3 \times 2 \times \text{截数}}{48 \times 7 \times \text{径}} \right) \\ & \quad + \left(\frac{15 \times \text{截数}}{384 \times \text{径}} - \frac{15 \times 2 \times \text{截数}}{384 \times 9 \times \text{径}} \right) \\ & = \frac{\text{截数}}{\text{径}} + \frac{\text{截数}}{2 \times 3 \times \text{径}} + \frac{3 \times \text{截数}}{8 \times 5 \times \text{径}} + \frac{3 \times 5 \times \text{截数}}{48 \times 7 \times \text{径}} + \frac{15 \times 7 \times \text{截数}}{384 \times 9 \times \text{径}} \end{aligned}$$

$$\frac{\text{截数}}{\text{径}} + \frac{\text{截数}}{2 \times 3 \times \text{径}} + \frac{3 \times \text{截数}}{8 \times 5 \times \text{径}} + \frac{15 \times \text{截数}}{48 \times 7 \times \text{径}} + \frac{105 \times \text{截数}}{384 \times 9 \times \text{径}}$$

これは、 $\frac{1}{\text{某甲}}$ 畳数 と全く同じなので

$$\frac{2 \times \text{円積率} \times \text{截数}}{\text{径}} = \frac{1}{\text{某甲}} \text{畳数} \quad \dots (28)$$

$$\frac{1}{\text{某甲}} \text{畳数} - \frac{\text{截数} \times \text{前空数}}{\text{径}} \quad \text{これを解いて通分すると}$$

$$\begin{aligned} & \frac{\text{截数}}{\text{径}} + \frac{\text{截数}}{2 \times 3 \times \text{径}} + \frac{3 \times \text{截数}}{8 \times 5 \times \text{径}} + \frac{15 \times \text{截数}}{48 \times 7 \times \text{径}} + \frac{105 \times \text{截数}}{384 \times 9 \times \text{径}} \\ & - \frac{\text{截数}}{\text{径}} \times \left(1 - \frac{1}{2} - \frac{1}{8} - \frac{3}{48} - \frac{15}{384} \right) \end{aligned}$$

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$$\begin{aligned}
 &= \left(\frac{\text{截数}}{2 \times 3 \times \text{径}} + \frac{\text{截数}}{2 \times \text{径}} \right) + \left(\frac{3 \times \text{截数}}{8 \times 5 \times \text{径}} + \frac{\text{截数}}{8 \times \text{径}} \right) + \left(\frac{15 \times \text{截数}}{48 \times 7 \times \text{径}} + \frac{3 \times \text{截数}}{48 \times \text{径}} \right) \\
 &\quad + \left(\frac{105 \times \text{截数}}{384 \times 9 \times \text{径}} + \frac{15 \times \text{截数}}{384 \times \text{径}} \right) \\
 &= \frac{4 \times \text{截数}}{2 \times 3 \times \text{径}} + \frac{3 \times \text{截数} + 5 \times \text{截数}}{8 \times 5 \times \text{径}} + \frac{15 \times \text{截数} + 21 \times \text{截数}}{48 \times 7 \times \text{径}} \\
 &\quad + \frac{105 \times \text{截数} + 135 \times \text{截数}}{384 \times 9 \times \text{径}} \\
 &= \frac{2 \times \text{截数}}{3 \times \text{径}} + \frac{2 \times \text{截数}}{2 \times 5 \times \text{径}} + \frac{2 \times 3 \times \text{截数}}{8 \times 7 \times \text{径}} + \frac{2 \times 15 \times \text{截数}}{48 \times 9 \times \text{径}}
 \end{aligned}$$

$$0 + \frac{2 \times \text{截数}}{3 \times \text{径}} + \frac{2 \times \text{截数}}{2 \times 5 \times \text{径}} + \frac{2 \times 3 \times \text{截数}}{8 \times 7 \times \text{径}} + \frac{2 \times 15 \times \text{截数}}{48 \times 9 \times \text{径}}$$

これは、 $\frac{\text{天}^2}{\text{某甲}}$ 畳数 の2倍と全く同じなので

$$\frac{\text{円積率} \times \text{截数}}{\text{径}} = \frac{\text{天}^2}{\text{某甲}} \text{畳数} \quad \dots (29)$$

$\frac{3 \times \text{天}^2}{\text{某甲}} \text{畳数} - \frac{\text{截数} \times \text{前空数}}{\text{径}}$ これを解いて通分すると

$$\begin{aligned}
 &\frac{3 \times \text{截数}}{3 \times \text{径}} + \frac{3 \times \text{截数}}{2 \times 5 \times \text{径}} + \frac{3 \times 3 \times \text{截数}}{8 \times 7 \times \text{径}} + \frac{3 \times 15 \times \text{截数}}{48 \times 9 \times \text{径}} + \frac{3 \times 105 \times \text{截数}}{384 \times 11 \times \text{径}} \\
 &- \frac{\text{截数}}{\text{径}} \times \left(1 - \frac{1}{2} - \frac{1}{8} - \frac{3}{48} - \frac{15}{384} \right) \\
 &= \left(\frac{3 \times \text{截数}}{2 \times 5 \times \text{径}} + \frac{\text{截数}}{2 \times \text{径}} \right) + \left(\frac{3 \times 3 \times \text{截数}}{8 \times 7 \times \text{径}} + \frac{\text{截数}}{8 \times \text{径}} \right) + \left(\frac{3 \times 15 \times \text{截数}}{48 \times 9 \times \text{径}} + \frac{3 \times \text{截数}}{48 \times \text{径}} \right) \\
 &\quad + \left(\frac{3 \times 105 \times \text{截数}}{384 \times 11 \times \text{径}} + \frac{15 \times \text{截数}}{384 \times \text{径}} \right) \\
 &= \frac{8 \times \text{截数}}{2 \times 5 \times \text{径}} + \frac{9 \times \text{截数} + 7 \times \text{截数}}{8 \times 7 \times \text{径}} + \frac{45 \times \text{截数} + 27 \times \text{截数}}{48 \times 9 \times \text{径}} \\
 &\quad + \frac{315 \times \text{截数} + 165 \times \text{截数}}{384 \times 11 \times \text{径}}
 \end{aligned}$$

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$$= \frac{4 \times \text{截数}}{5 \times \text{径}} + \frac{4 \times \text{截数}}{2 \times 7 \times \text{径}} + \frac{4 \times 3 \times \text{截数}}{8 \times 9 \times \text{径}} + \frac{4 \times 15 \times \text{截数}}{48 \times 11 \times \text{径}}$$

$$0 + \frac{4 \times \text{截数}}{5 \times \text{径}} + \frac{4 \times \text{截数}}{2 \times 7 \times \text{径}} + \frac{4 \times 3 \times \text{截数}}{8 \times 9 \times \text{径}} + \frac{4 \times 15 \times \text{截数}}{48 \times 11 \times \text{径}}$$

これは、 $\frac{\text{天}^4}{\text{某甲}}$ 量数 の4倍と全く同じなので

$$\frac{3 \times \text{円積率} \times \text{截数}}{4 \times \text{径}} = \frac{\text{天}^4}{\text{某甲}} \text{量数} \quad \dots (30)$$

$$\frac{5 \times \text{天}^4}{\text{某甲}} \text{量数} - \frac{\text{截数} \times \text{前空数}}{\text{径}} \quad \text{これを解いて通分すると}$$

$$\begin{aligned} & \frac{5 \times \text{截数}}{5 \times \text{径}} + \frac{5 \times \text{截数}}{2 \times 7 \times \text{径}} + \frac{5 \times 3 \times \text{截数}}{8 \times 9 \times \text{径}} + \frac{5 \times 15 \times \text{截数}}{48 \times 11 \times \text{径}} + \frac{5 \times 105 \times \text{截数}}{384 \times 13 \times \text{径}} \\ & - \frac{\text{截数}}{\text{径}} \times \left(1 - \frac{1}{2} - \frac{1}{8} - \frac{3}{48} - \frac{15}{384} \right) \\ & = \left(\frac{5 \times \text{截数}}{2 \times 7 \times \text{径}} + \frac{\text{截数}}{2 \times \text{径}} \right) + \left(\frac{5 \times 3 \times \text{截数}}{8 \times 9 \times \text{径}} + \frac{\text{截数}}{8 \times \text{径}} \right) + \left(\frac{5 \times 15 \times \text{截数}}{48 \times 11 \times \text{径}} + \frac{3 \times \text{截数}}{48 \times \text{径}} \right) \\ & \quad + \left(\frac{5 \times 105 \times \text{截数}}{384 \times 13 \times \text{径}} + \frac{15 \times \text{截数}}{384 \times \text{径}} \right) \\ & = \frac{12 \times \text{截数}}{2 \times 7 \times \text{径}} + \frac{15 \times \text{截数} + 9 \times \text{截数}}{8 \times 9 \times \text{径}} + \frac{75 \times \text{截数} + 33 \times \text{截数}}{48 \times 11 \times \text{径}} \\ & \quad + \frac{525 \times \text{截数} + 195 \times \text{截数}}{384 \times 13 \times \text{径}} \\ & = \frac{6 \times \text{截数}}{7 \times \text{径}} + \frac{6 \times \text{截数}}{2 \times 9 \times \text{径}} + \frac{6 \times 3 \times \text{截数}}{8 \times 11 \times \text{径}} + \frac{6 \times 15 \times \text{截数}}{48 \times 13 \times \text{径}} \end{aligned}$$

$$0 + \frac{6 \times \text{截数}}{7 \times \text{径}} + \frac{6 \times \text{截数}}{2 \times 9 \times \text{径}} + \frac{6 \times 3 \times \text{截数}}{8 \times 11 \times \text{径}} + \frac{6 \times 15 \times \text{截数}}{48 \times 13 \times \text{径}}$$

これは、 $\frac{\text{天}^6}{\text{某甲}}$ 量数 の6倍と全く同じなので

$$\frac{5 \times 3 \times \text{円積率} \times \text{截数}}{6 \times 4 \times \text{径}} = \frac{\text{天}^6}{\text{某甲}} \text{疊数} \quad \dots (31)$$

$$\frac{7 \times \text{天}^6}{\text{某甲}} \text{疊数} - \frac{\text{截数} \times \text{前空数}}{\text{径}} \quad \text{これを解いて通分すると}$$

$$\begin{aligned} & \frac{7 \times \text{截数}}{7 \times \text{径}} + \frac{7 \times \text{截数}}{2 \times 9 \times \text{径}} + \frac{7 \times 3 \times \text{截数}}{8 \times 11 \times \text{径}} + \frac{7 \times 15 \times \text{截数}}{48 \times 13 \times \text{径}} + \frac{7 \times 105 \times \text{截数}}{384 \times 15 \times \text{径}} \\ & - \frac{\text{截数}}{\text{径}} \times \left(1 - \frac{1}{2} - \frac{1}{8} - \frac{3}{48} - \frac{15}{384} \right) \\ & = \left(\frac{7 \times \text{截数}}{2 \times 9 \times \text{径}} + \frac{\text{截数}}{2 \times \text{径}} \right) + \left(\frac{7 \times 3 \times \text{截数}}{8 \times 11 \times \text{径}} + \frac{\text{截数}}{8 \times \text{径}} \right) + \left(\frac{7 \times 15 \times \text{截数}}{48 \times 13 \times \text{径}} + \frac{3 \times \text{截数}}{48 \times \text{径}} \right) \\ & \quad + \left(\frac{7 \times 105 \times \text{截数}}{384 \times 15 \times \text{径}} + \frac{15 \times \text{截数}}{384 \times \text{径}} \right) \\ & = \frac{16 \times \text{截数}}{2 \times 9 \times \text{径}} + \frac{21 \times \text{截数} + 11 \times \text{截数}}{8 \times 11 \times \text{径}} + \frac{105 \times \text{截数} + 39 \times \text{截数}}{48 \times 13 \times \text{径}} \\ & \quad + \frac{735 \times \text{截数} + 225 \times \text{截数}}{384 \times 15 \times \text{径}} \\ & = \frac{8 \times \text{截数}}{9 \times \text{径}} + \frac{8 \times \text{截数}}{2 \times 11 \times \text{径}} + \frac{8 \times 3 \times \text{截数}}{8 \times 13 \times \text{径}} + \frac{8 \times 15 \times \text{截数}}{48 \times 15 \times \text{径}} \end{aligned}$$

$$0 + \frac{8 \times \text{截数}}{9 \times \text{径}} + \frac{8 \times \text{截数}}{2 \times 11 \times \text{径}} + \frac{8 \times 3 \times \text{截数}}{8 \times 13 \times \text{径}} + \frac{8 \times 15 \times \text{截数}}{48 \times 15 \times \text{径}}$$

これは、 $\frac{\text{天}^8}{\text{某甲}} \text{疊数}$ の8倍と全く同じなので

$$\frac{7 \times 5 \times 3 \times \text{円積率} \times \text{截数}}{8 \times 6 \times 4 \times \text{径}} = \frac{\text{天}^6}{\text{某甲}} \text{疊数} \quad \dots (32)$$

このように、求めて、甲除偶乗表 を作る